Phonon avalanches in paramagnetic impurities with spin $S=\frac{1}{2}$

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(Received 19 December 2002; published 23 June 2003)

We theoretically study the dynamics of transverse-and-longitudinal acoustic waves propagating parallel to an external magnetic field in a crystal containing ion impurities with an effective spin $S=\frac{1}{2}$. Corresponding evolution equations describing the coherent pulse evolution are derived. These equations are used to study the phonon avalanches arising due to decay of an initially unstable state of the spin systems for different geometries of interaction. It is found that the coherent dynamics of acoustic pulses propagated in one direction is described by a pair of integrable systems of evolution equations. The picosecond acoustic pulses governed by these systems are "a few-cycle" pulses. By using a modified set of equations of the inverse scattering transform, it is found that the strong interaction of three or two components of the acoustic waves with the spin system is asymptotically described by the quasi-self-similar solutions. Physical applications of the obtained results are discussed.

DOI: 10.1103/PhysRevE.67.066606 PACS number(s): 41.20.Jb, 42.50.Md, 43.25.+y

I. INTRODUCTION

Generation of picosecond acoustic pulses under laboratory conditions [1,2] gave rise to a number of theoretical papers dedicated to the interaction of such pulses with paramagnetic crystals. Evolution of ultrashort and a few-cycle pulses is described by complex systems of partial equations as usual. In some cases, nonlinear coherent processes associated with such pulses can be described in the framework of integrable evolution models. Application of the inverse scattering transform (IST) to such models in optical systems allowed one to obtain the most detailed information about the evolution of system [3,4]. For a few-cycle optical pulses, the IST was applied to the reduced Maxwell-Bloch equations in Ref. [5] and to its generalization in Ref. [6] to find a set of soliton solutions.

Evolution of picosecond acoustic pulses attracts a special attention because these pulses correspond to the length $\sim 10^{-7}-10^{-6}$ cm and to the highest pick power. These properties are very perspective for diagnostics, nonlinear acousto-optical processes, and so on. Time scale of picosecond acoustic pulses corresponds to a few-cycle pulses, i.e., an approximation of the slow changing amplitudes and phases cannot be applied.

As a physical realization of the model of acoustic pulses evolution, the crystal MgO containing Kramers's doublets impurities of the paramagnetic ions Co^{2+} may be proposed [7–9]. Theoretical papers devoted to the study of dynamics of the coherent acoustic pulses in paramagnetic with $S=1/2$ impurities and Zeeman splitting used as a rule an analogy with known optical two-level systems. Theory of the quasimonochromatic had been developed, for instance, in Ref. [10] and for a few cycle acoustic pulses in Ref. [11]. In the later papers, approximations used by the authors are not valid for a few-cycle pulses. Another extreme case considered in Ref. [11] corresponds to the extremely short durations of the transverse acoustic pulses, i.e., durations of pulses are much more shorter than a period of oscillation. However, this approximation requires unrealistic physical conditions.

At the same time, the rich structure of evolution equations describing acoustic pulses evolution opens up the possibility of reducing them, for quite realistic approximations, to integrable models when similar stringent conditions are imposed and also without them.

This paper is concerned with development in the theoretical study of avalanches of acoustic phonons from stimulated emission by population-inverted spin system. Recently, avalanches of resonant acoustic phonons are observed following population inversion of the Zeeman split $\overline{E}(^2E)$ doublet in dilute ruby by selective optical pumping, see Refs. [13,14], and references therein. For description of observed time dependence of levels populations, coherent equations of motion for the lattice displacement and spin Bloch vector had been used.

In this paper, we derive the evolution equations describing propagation of the transverse-longitudinal acoustic waves in crystal containing ion impurities with an effective spin 1/2 for the different geometries of interaction. Using an approximation of one-directional propagation, we find integrable reductions of these equations and develop a corresponding technique of the IST for two different geometries of interaction. The derived integrable models describe propagation of a few-cycle acoustic pulses without using a slow envelope and low amplitude approximations. The IST technique is used to find solution for the leading front of solutions described by the avalanches of phonons. We also find that in quasimonochromatic approximations, obtained systems of evolution equations become formally equivalent and reduced to integrable equations analogous to the Maxwell-Bloch equations for a two-level system containing a quadratic Stark frequency shift.

II. PHYSICAL ONSET OF THE MODELS

Consider a system of ions with spin $S=1/2$ implemented in a crystal. Assume that an external constant and uniform

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magnetic field **B** is directed along the z axis. We assume that a strain pulse propagates along the z axis as well. Zeeman interaction of the magnetic moment $\hat{\mu}^{(a)}$ at a point a contribute $\hat{H}_a = -\hat{\mu}^{(a)}\mathbf{B}$ to a total Hamiltonian. The $\hat{\mu}^{(a)}$ components can be expressed in terms of spin $\mathbf{S}^{(a)}(\mathbf{r}_a)$ components as $\hat{\mu}_j^{(a)} = -\Sigma_k \mu_B g_{jk}\hat{S}_k^{(a)}$, where $\mathbf{r}_a$ is the radius vector of spin at the point a, μ_B is the Bohr magneton, g_{jk} are the components of the Lande tensor.

The Hamiltonian describing dynamics of the spins in a crystal takes the form

$$\hat{H}^z = \sum_{a=1}^{N} \hat{H}_a^z = \mu_B \sum_a \sum_{j,k} B_j g_{jk}\hat{S}_k^{(a)}. \tag{1}$$

Since the effective spin is 1/2, it can be decomposed into Pauli matrices:

$$\hat{S}_x^a = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \hat{S}_y^a = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \hat{S}_z^a = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2}$$

Assume that the x,y,z coordinates along the principal Lande tensor axes coincide with the crystal symmetry axes. The Lande tensor is then diagonal in a nondeformed unperturbed medium: $g_{jk} = g_{jk}^{(0)} = g_{jj}\delta_{jk}$, where δ_{jk} is the delta function. The deformation of the crystal by an acoustic wave is described by linear corrections to the Lande tensor [12]

$$g_{jk} = g_{jk}^{(0)} + \sum_{p,q}\left(\frac{\partial g_{jk}}{\partial \mathcal{E}_{pq}}\right)_0 \mathcal{E}_{pq} + \cdots, \tag{3}$$

where $\mathcal{E}_{pq}$ are the components of the strain tensor of crystal. The derivatives are taken at the point of zero deformation. The strain tensor components can be expressed in term of the components of displacement vector $\mathbf{U} = (U_x, U_y, U_z)$ as [12]: $\mathcal{E}_{pq} = \frac{1}{2}(\partial U_p/\partial x_q + \partial U_q/\partial x_p)$.

Term containing the first degree of the strain tensor $\mathcal{E}_{pq}$ describes the spin-phonon interaction contribution to the total Hamiltonian

$$\hat{H}_{int} = \sum_\alpha \sum_{j,k,p,q} \mu_B B_j F_{jk,pq}\mathcal{E}_{pq}\hat{S}_k^{(a)}, \tag{4}$$

where $F_{jk,pq} = (\partial g_{jk}/\partial \mathcal{E}_{pq})_0$ are the spin-phonon coupling constants [8].

We use quasiclassical description of the spin-phonon interaction, i.e., acoustic fields (components of the strain tensor) are classical but spins are treated as a quantum system. Under such conditions, we derive the following contribution to the Hamiltonians associated with the impurities:

$$\hat{H}_s = \int n\hbar\omega_B S_z d^3\mathbf{r}, \tag{5}$$

$$\langle \hat{H}_{int}\rangle = \sum_\alpha \sum_{j,k,p,q} \mu_B B_j F_{jk,pq} \int \mathcal{E}_{pq}(\mathbf{r})\langle \hat{S}_k^{(a)}(\mathbf{r})\rangle d^3\mathbf{r}, \tag{6}$$

here and below in this paper index γ has the meanings: x,y,z. $S_\gamma = \mathrm{Tr}\{\hat{s}_\gamma\hat{\rho}\}/2$, $\hat{\rho}$ is the density matrix, $\hat{s}_\gamma$ are the Pauli matrices, i.e., $S_x = (\rho_{12}+\rho_{21})/2$, $S_y = i(\rho_{12}-\rho_{21})/2$, $S_z = (\rho_{11}-\rho_{22})/2$, ρ_{ij} are the components of $\hat{\rho}$. $\omega_B = g\mu_B B/\hbar$ is the frequency of the Zeeman splitting of the Kramers's doublets, $B = |\mathbf{B}|$, and $g = g_{xx} = g_{yy} = g_{zz}$. $n(\mathbf{r}) = \Sigma_j \delta(\mathbf{r}-\mathbf{r}_j)$ is the density of the paramagnetic impurities, $\delta(\mathbf{r})$ is the delta-function. The integrals are taken over the crystal volume. Angular brackets denote that the Hamiltonian is averaged over the quantum states.

Wave dynamics of an acoustic wave in a crystal without anharmonicity is described by the Hamiltonian

$$\hat{H}_a = \frac{1}{2}\int \sum_{\gamma=x,z}\left\{\frac{p_\gamma^2}{n_0} + \lambda_\gamma\left(\frac{\partial U_\gamma}{\partial z}\right)^2\right\} d^3\mathbf{r}, \tag{7}$$

where n_0 is the mean crystal density, p_γ are the momentum density components, associated with the dynamic displacements, and λ_γ are the elements of the elastic module of a crystal [8]. We assume that the number of phonons is large and that the classical description for acoustic fields dynamics is valid.

Evolution equation of an effective spin and field (components of the strain tensor) are

$$i\hbar\frac{\partial\hat{\rho}}{\partial t} = [\hat{H}, \hat{\rho}], \tag{8}$$

$$\frac{\partial \mathbf{U}}{\partial t} = \frac{\partial H}{\partial \mathbf{p}}, \quad \frac{\partial \mathbf{p}}{\partial t} = -\frac{\partial H}{\partial \mathbf{U}}, \tag{9}$$

where $H = H_a + \langle \hat{H}_{int}\rangle$. We assume that the time scale is short enough to neglect relaxation effects for the acoustic pulses and spins in crystal. We consider only one-dimensional evolution of the fields along the z axis.

In a common case, evolution equations are very complex to be solved. Application of the symmetry of crystal and a particular choice of direction of the magnetic field can yield the essential simplifications of equations. The two particular cases of geometry of crystal and interaction are considered here. In both these cases, we derive the integrable systems of evolution equations, describing dynamics of the coherent a few-cycle acoustic pulses.

III. THREE-COMPONENT ACOUSTIC FIELD

In this section, we derive the evolution equations describing propagation of the transverse-longitudinal acoustic waves in a crystal containing ion impurities with an effective spin 1/2, assuming that a strain pulse propagates along the z axis parallel to an external constant magnetic field **B**. Let one of the axes of crystal having the fourth-order symmetry is directed along the z axis. Then, Hamiltonian of spin-phonon interaction takes the form

$$\langle \hat{H}_{int}\rangle = \int \sum_\gamma \frac{n\hbar\omega_B}{g} f_\gamma \mathcal{E}_{\gamma z} S_\gamma d^3\mathbf{r}, \tag{10}$$

here $\gamma = x,y,z$, $f_\gamma = (\partial g_{zz}/\partial \mathcal{E}_{\gamma z})$ are the coupling constants of the spin-phonon interaction [8]. Let $f_x = f_y$ due to assumed symmetry restrictions. However, $f_y, f_x \neq f_z$ in common.

Using Eqs. (7)–(10), we derive the following system of evolution equations:

$$\frac{\partial^2 \mathcal{E}_{\gamma z}}{\partial t^2} - v_\gamma^2 \frac{\partial^2 \mathcal{E}_{\gamma z}}{\partial z^2} = \frac{nG_\gamma}{n_0}\frac{\partial^2 S_\gamma}{\partial z^2}, \tag{11}$$

$$\frac{\partial S_x}{\partial t} = -\left(\omega_B + \frac{G_z \mathcal{E}_{zz}}{\hbar}\right) S_y + \frac{G_y \mathcal{E}_{yz}}{\hbar} S_z, \tag{12}$$

$$\frac{\partial S_y}{\partial t} = \left(\omega_B + \frac{G_z \mathcal{E}_{zz}}{\hbar}\right) S_x - \frac{G_x \mathcal{E}_{xz}}{\hbar} S_z, \tag{13}$$

$$\frac{\partial S_z}{\partial t} = \frac{1}{\hbar}(G_x \mathcal{E}_{xz} S_y - G_y \mathcal{E}_{yz} S_x), \tag{14}$$

where $G_\gamma = \hbar \omega_B f_\gamma / g$, $v_\gamma = \sqrt{\lambda_\gamma / n_0}$. The following normalized relation holds:

$$S_z^2 + S_x^2 + S_y^2 = (\rho_{11} + \rho_{22})^2 \equiv 1. \tag{15}$$

To derive an integrable reduction of system (11)–(14), we assume that the phase velocities of the components of acoustic wave to be equal: $v_x = v_y = v_z = v$. This assumption may be valid in crystal with central symmetric interaction, for instance, in ion crystal of halogenide of alkaline metals [8].

More often than not density of the paramagnetic impurities is relatively small in real crystal. Under this condition one can use an approximation of one-directional propagation of waves. This approximation is similar to the approximation used by the authors of Ref. [5] to derive the reduced Maxwell-Bloch equation for a two-level optical system. This approximation formally corresponds to the approximate equality $\partial_z \approx -v^{-1}\partial_t + O(\epsilon)$, here ϵ is a small parameter. This means that normalized density of impurities in crystal is of the same smallness as the derivative $\partial_{\tilde{\chi}} = \partial_z + v^{-1}\partial_t$ of the acoustic field $\mathcal{E}_\gamma$ amplitude. Then, we can replace the derivatives with respect to z on the right-hand side (RHS) of Eq. (17) for the derivatives $v^{-1}\partial_t$ with an accuracy $O(\epsilon^2)$. Thus, the condition of unidirectional pulse propagation is satisfied and one obtains the following equations:

$$\frac{\partial \mathcal{E}_\perp}{\partial \tilde{\chi}} = \frac{nG_x}{2v^2 n_0}\frac{\partial S_\perp}{\partial t}, \tag{16}$$

$$\frac{\partial \mathcal{E}_{zz}}{\partial \tilde{\chi}} = \frac{nG_z}{2v^2 n_0}\frac{\partial S_z}{\partial t}, \tag{17}$$

where $S_\perp = \rho_{21}$, $E_\perp = \mathcal{E}_{xz} + i\mathcal{E}_{yz}$.

This approximation does not impose any restrictions to duration of the acoustic pulses and can be used for the pulse duration $\sim \omega_B^{-1}$, i.e., for a few-cycle pulses.

Then from Eqs. (12)–(17), one can derive the simple relation

$$|E_\perp|^2 + \left(\mathcal{E}_{zz} + \frac{\omega_B \hbar}{G_z}\right)^2 = U_0^2(t). \tag{18}$$

Here a real function $U_0(t)$ is determined by the boundary conditions.

System (12)–(17) is the integrable system of evolution equations. Using Eq. (23), we rewrite this system in the form

$$\partial_\chi E = i b_0 U S_\perp - i E S_z, \tag{19}$$

$$\partial_\tau S_\perp = i b_0 U S_\perp - i E S_z, \tag{20}$$

$$\partial_\tau S_z = \frac{i}{2}(E S_\perp^* - E^* S_\perp), \tag{21}$$

where $U^2(\chi,\tau) = 1 - |E(\tau,\chi)|^2$ and

$$E = \frac{E_\perp}{U_0(t)}, \quad \chi = \tilde{\chi}\frac{nG_\perp^2}{2\hbar n_0 v^2}, \quad \tau = \int_0^t \frac{U_0(t')G_\perp}{\hbar}dt',$$

$$b_0 = \frac{G_\parallel}{G_\perp}.$$

IV. THE ISTM EQUATIONS FOR THREE-COMPONENT ACOUSTIC WAVES

Let us solve the problem for the fast enough decaying potential $E(\tau,\chi) \to 0$ and its derivatives with respect to τ for $\tau \to \pm\infty$. System (19)–(21) can be presented as the compatibility condition of the following pair of linear systems:

$$\partial_\tau \Phi = \begin{pmatrix} -i\lambda U & (\lambda+\beta)E \\ -\lambda E^* & i\lambda U \end{pmatrix}\Phi, \tag{22}$$

$$\partial_\chi \Phi = \frac{1}{(2\lambda + b_0)}\begin{pmatrix} i\lambda S_z & b_0(\lambda+\beta)S_\perp \\ -b_0\lambda S_\perp^* & -i\lambda S_z \end{pmatrix}\Phi, \tag{23}$$

where λ is a spectral parameter, $U^2 + EE^* = 1$, $\beta = \frac{1}{2}(b_0 - b_0^{-1})$, $b_0 \neq 0$. Note that the spectral problem (22) differs from the related problems (studied here) associated with the solution of the Heisenberg and Landau-Lifshitz equations or equations of Raman scattering, see Refs. [15–17], by its symmetry properties. Therefore, the IST apparatus must be developed for this and models presented below by taking into account their specifics. We present here only main steps.

Spectral problem (22) possess the involution properties

$$\Phi = \hat{M}\Phi(\lambda^*)^*\hat{M}^{-1}, \tag{24}$$

where

$$\hat{M} = \begin{pmatrix} 0 & (\lambda+\beta)/\lambda \\ -1 & 0 \end{pmatrix}. \tag{25}$$

Introduce the Jost functions $\Psi^\pm$, the solutions of Eq. (22) with the asymptotic:

$$\Psi^\pm = \exp(-i\lambda\sigma_3\tau), \quad \tau \to \pm\infty. \tag{26}$$

The symmetry property (24) corresponds to the following matrix form of the Jost functions:

$$\Psi^{\pm}=\begin{pmatrix} \psi_1^{\pm} & -\psi_2^{\pm *}\dfrac{\lambda+\beta}{\lambda} \\ \psi_2^{\pm} & \psi_1^{\pm *} \end{pmatrix}. \tag{27}$$

These solutions are related by the scattering matrix $\hat{T}$ by the relation

$$\Psi^{-}=\Psi^{+}\hat{T}, \tag{28}$$

which can be chosen in the form

$$\hat{T}=\begin{pmatrix} a^* & b(\lambda+\beta)/\lambda \\ -b^* & a \end{pmatrix}. \tag{29}$$

The Jost functions have standard analytical properties. The function $a(\lambda)$ is holomorphic in the upper half plane λ, where its zeros correspond to the soliton solutions [18].

Let us substitute the following integral representation of the Jost functions:

$$\Psi^{+}(\tau)=e^{-i\lambda\sigma_3\tau}+\int_{-\infty}^{\tau}\begin{pmatrix} \lambda K_1(\tau,s) & (\lambda+\beta)K_2(\tau,s) \\ -\lambda K_2^*(\tau,s) & \lambda K_1^*(\tau,s) \end{pmatrix} e^{-i\lambda\sigma_3 s}ds \tag{30}$$

into Eq. (28) and integrating from $-\infty$ to ∞ over λ with the weight $e^{-i\lambda y}(2\pi\lambda)^{-1}$. As a result, we obtain the following Marchenko-type equations:

$$K_2^*(\tau,y)=F_0(\tau+y)+i\int_{-\infty}^{\tau}K_1(\tau,s)\partial_y F_0(s+y)ds, \quad y\leqslant\tau \tag{31}$$

$$K_1^*(\tau,y)=-\int_{-\infty}^{\tau}K_2(\tau,s)(\beta+i\partial_y)F_0(s+y)ds, \quad y\leqslant\tau. \tag{32}$$

Where

$$F_0(y)=\int_{\mathcal{C}}\frac{b(\chi)}{a(\chi)}e^{-i\lambda y}\frac{d\lambda}{2\pi\lambda}, \tag{33}$$

$\mathcal{C}$ is the contour along the real axis and that passes above all poles in the upper half plane of the complex plane of λ. a,b consists a part of the spectral data, determined, in common, by the initial-boundary conditions.

Substitute function (30) in the spectral problem (22), we derive the relation between potential and the kernels $K_{1,2}$ in the form

$$E(\tau)=\frac{2[1-iK_1(\tau,\tau)]K_2^*(\tau,\tau)}{[1+iK_1^*(\tau,\tau)][1-iK_1(\tau,\tau)]+|K_2(\tau,\tau)|^2}. \tag{34}$$

Let us study the dynamics of the acoustic phonons avalanche arising for an initially completely inverted spin system and a small initial acoustical noise. The initial-boundary conditions for such state can be written in the following form:

$$|E(\tau,0)|=\text{const}\ll 1, \quad S_z(-\infty,\chi)=1, \quad S_\perp(-\infty,\chi)=0, \tag{35}$$

where the acoustical noise is modeled by $E(\tau,0)$. For this initial-boundary conditions, only the real spectrum gives contribution to solution. The dependence of the scattering coefficient $R=b/a$ by χ can be found by means of a standard way [3]. For Eq. (35), we find

$$R(\chi)=\frac{b(\chi;\lambda)}{a(\chi;\lambda)}=\rho_0\exp\left(\frac{-2i\chi\lambda}{2\lambda+b_0}\right). \tag{36}$$

Next we have to substitute the kernel $F_0(y,\chi)$ and its derivative

$$i\partial_y F_0(\tau+y;\chi)=\frac{\rho_0}{2\pi}\int_{\mathcal{C}_1}e^{-i\lambda(\tau+y)+i\chi b_0/2\lambda+i[(\tau+y)b_0]/2-i\chi}d\lambda, \tag{37}$$

$$F_0^*(\tau+y;\chi)=\frac{\rho_0}{2\pi}\int_{\mathcal{C}_2}e^{i\mu(\tau+y)+i\chi b_0/2\mu+i[(\tau+y)b_0]/2-i\chi}\frac{d\mu}{\mu-\dfrac{b_0}{2}} \tag{38}$$

in the Marchenko equations (31) and (32). Then change integration variables $\lambda=\sqrt{[\chi b_0/2(\tau+y)]}\tilde{\lambda}$ ($\mu=\sqrt{[\chi b_0/2(\tau+y)]}\tilde{\mu}$) and deform the contours of integrations $\mathcal{C}_1$ ($\mathcal{C}_2$) in such a way that they bend around the point $\tilde{\lambda}=i$ ($\tilde{\mu}=-i$) in the positive (negative) directions. The main contributions to the integrals arise from the exponents, therefore, we can approximately replace $\sqrt{y+\tau}\approx\sqrt{2\tau}$, $\sqrt{s+\tau}\approx\sqrt{2\tau}$, $1+O(b_0\chi\tau)^{-1}\approx 1$ in the exponential factors. Let us introduce the new functions

$$Q_{1,2}(\tau)=\int_{-\infty}^{\tau}\frac{\sqrt{b_0\chi}}{4\sqrt{\tau}}K_{1,2}(\tau,s)\exp\left[\sqrt{2\chi b_0(\tau+s)}+is\frac{b_0}{2}\right]ds. \tag{39}$$

Equations (31) and (32) become algebraic ones for $Q_{1,2}(\tau)$ and can be easily solved.

Using these solutions one easily obtains the kernels $K_{1,2}$ and using them, we find the solution for the acoustic field amplitude

$$E(\tau,\chi)=\frac{2Z\rho_0 e^{\theta+i\tau b_0/2}}{Z^2+|\rho_0|^2e^{2\theta}}\left[1+O\left(\frac{1}{\sqrt{\chi}}\right)\right], \tag{40}$$

where

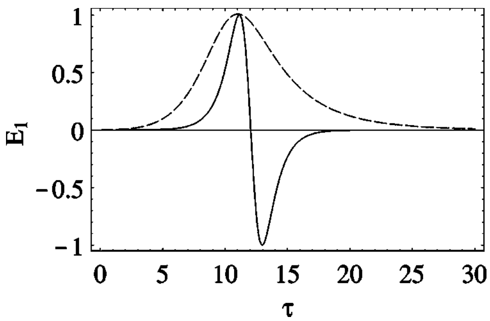

FIG. 1. Dependence of $E_1 = Ee^{-ib_0\tau/2}$ versus τ. Solution (40) is shown by the solid line. Numerical solution of the Maxwell-Bloch equations normalized to unity is shown by the dashed line.

$$\theta = 2\sqrt{b_0\chi\tau}, \quad Z = 1 - \frac{|\rho_0^2|}{4}e^{2\theta}.$$

This solution describes the first pulse of an infinite sequences of nonlinear pulsations with decaying amplitudes and increasing widths. Relaxation, diffraction, inhomogeneity of initial inversion additionally reduce amplitudes of the second and following pulses in comparison with that of the first one. Therefore, as a rule contributions of these pulses except the first one can be neglected. Solution (40) up to the linear phase is approximately self-similar, i.e., this solution depends on θ with the accuracy $O(1/\sqrt{\chi})$. In Ref. [13] a known solution to the Maxwell-Bloch equations was used for description of the phonon avalanche for the transverse acoustic waves. Analogy of the avalanche and optical superradiance effect arising in initially inverted two-level system was used. An evolution of transverse-and-longitudinal acoustic wave described by model (19)–(21) may significantly differ from that of a transverse field, see Ref. [13]. For instance, an amplitude of field in the avalanche regime described by the solution of the Maxwell-Bloch system increases with χ as $\sim\sqrt{\chi S_0}$, where S_0 is the initial density of inverted spins. On the other hand, the maximum value of the transverse-and-longitudinal acoustic wave is restricted by $|U_0|$. Leading pulses of the trains of pulses corresponding to both waves are shown in Fig. 1. The dashed line shows numerical calculation of the amplitude of the leading pulse of transverse field obtained by the numerical solution of the Maxwell-Bloch equations. Dependence of E versus variable τ described by solution (40) is depicted in Fig. 1 by the solid line. Parameters of this wave are chosen such that the maximum of its amplitude equals to U_0. Deriving solution (40), we neglect terms having $O(1/\sqrt{\chi})$. It may be shown that term of such order yields the nonlinear phase modulation, i.e., nonlinear rotation of the pulse polarization with the coefficient $\sim\beta/\sqrt{\chi}$. This rotation is a consequence of an asymmetry of the interaction arising from the deviation of b_0 from 1.

V. TWO-PARTIAL ACOUSTIC FIELD

Consider another geometry of interaction, corresponding to a physical situation then the contribution of the component of strain tensor $\mathcal{E}_{yz}$ may be neglected. This means that one may take into account only dynamics of acoustic wave described by the components of strain tensor $\mathcal{E}_{xz}$ and $\mathcal{E}_{zz}$. Such a situation can be realized in layered crystals in which the spin coupling with $\mathcal{E}_{yz}$ is relatively small. On the other hand, it is experimentally observed that in some crystals a group velocity of one of the transverse component of acoustic wave, e.g. v_y may significantly differ from the group velocities of another transverse-and-longitudinal components, such that $v_x \approx v_z$. As a consequence, the period of interaction of the y component of acoustic wave with the x and z components is relatively short and this interaction can be neglected, if one investigates only the dynamic of the x and z components of the acoustic field. More often than not such dynamics may be associated with an evolution in a thin layer or with surface waves [10]. The simplest surface waves are the two-partial Rayleigh waves or the surface shift waves [9], which consist of one transverse-and-longitudinal components of acoustic waves [9].

Using above assumptions, we restrict our investigation to dynamics of one transverse-and-longitudinal components of the acoustic waves: $\mathcal{E}_{xz}$, $\mathcal{E}_{zz}$. In this case only B_z, B_x (z and x components of $\mathbf{B}$) yield contributions to interaction. Assume that the vector of the magnetic field $\mathbf{B}$ lies in the xz plane. Under above assumptions spin-photon interaction is characterized by a following set of the coupling coefficients:

$$f_1 = \sum_{j=x,z} B_j g_{jz,xz} B^{-1}, \quad f_2 = \sum_{j=x,z} B_j g_{jz,zz} B^{-1},$$

$$f_3 = \sum_{j=x,z} B_j g_{jx,xz} B^{-1}, \quad f_4 = \sum_{j=x,z} B_j g_{jx,zz} B^{-1}. \quad (41)$$

The resulting Hamiltonians $\hat{H}_s$ and $\hat{H}_{int}$ take the forms

$$\hat{H}_s = \int \sum_\alpha n\hbar\omega_B \hat{S}_z^{(\alpha)} d^3\mathbf{r}, \quad (42)$$

$$\hat{H}_{int} = \int \sum_\alpha \frac{n\hbar\omega_B}{g}\{(f_1\mathcal{E}_{zz} + f_2\mathcal{E}_{xz})\hat{S}_z^{(\alpha)} + (f_3\mathcal{E}_{xz} + f_4\mathcal{E}_{zz})\hat{S}_x^{(\alpha)}\} d^3\mathbf{r}. \quad (43)$$

Introduce the effective transverse (or quasitransverse) acoustic field, i.e., linear combination of components of the strain tensor coupled in Hamiltonian with the spin component S_x,

$$\mathcal{W} = \mathcal{E}_{xz} + \frac{f_4}{f_3}\mathcal{E}_{zz}. \quad (44)$$

Then, we are able to rewrite sum of Hamiltonians (42) and (43) in the form

$$\hat{H}_s + \hat{H}_{int} = \int \frac{n\hbar\omega_B}{g}[\sigma_3(g + f\mathcal{E}_{zz}) + f_3\hat{\nu}\cdot\mathcal{W}] d^3\mathbf{r}, \quad (45)$$

where $f = f_2 - f_1 f_4/f_3$. The tensor coefficient before the last term on the RHS of Eq. (45) corresponds to the effective magnetic moment $\mu_B\hat{\nu}$,

$$\hat{\nu}=\begin{pmatrix} a & 1 \\ 1 & -a \end{pmatrix}, \tag{46}$$

where $a=f_1/f_3$ is an analog of the permanent dipole momentum known in the nonlinear optics, see, e.g., in Ref. [6]. We assume that $a=0$. Such a situation can be realized if electron-phonon couplings in a lower state and an upper state of two-level system are identical. Case of $f=0$ corresponds to a model that is formally equivalent to the integrable generalization of the reduced Maxwell-Bloch equations found recently by the authors of Ref. [6].

Using Eqs. (8) and (45), one derives the following Bloch equations for the effective two-level system:

$$\frac{\partial}{\partial t}S_z=\frac{f_3}{\hbar}\mathcal{W}S_y,$$

$$\frac{\partial}{\partial t}S_y=\left(\omega_B+\frac{f}{\hbar}\mathcal{E}_{zz}\right)S_x-\frac{f_3}{\hbar}\mathcal{W}S_z, \tag{47}$$

$$\frac{\partial}{\partial t}S_x=-\left(\omega_B+\frac{f}{\hbar}\mathcal{E}_{zz}\right)S_y,$$

here, as in the preceding section $S_\gamma=\mathrm{Tr}\,\hat{S}_\gamma^{(a)}\hat{\rho}/2,\ \gamma=x,y,z.$

Next we have to derive evolution equations for the fields $\mathcal{W},\mathcal{E}_{zz}$. For this aim, we assume that the phase velocities corresponding to the components of deformations U_x and U_z are equal to each other. Using equations for U_x, U_z, differentiating them with respect to z and using the definition of the strain tensor, we find that the classical fields obey the following evolution equations:

$$\frac{\partial^2\mathcal{W}}{\partial t^2}-v_1^2\frac{\partial^2\mathcal{W}}{\partial z^2}=\frac{2n\hbar\omega_B f_3}{gn_0}\frac{\partial^2 S_x}{\partial z^2}, \tag{48}$$

$$\frac{\partial^2\mathcal{E}_{zz}}{\partial t^2}-v_2^2\frac{\partial^2\mathcal{E}_{zz}}{\partial z^2}=\frac{2n\hbar\omega_B f}{gn_0}\frac{\partial^2 S_z}{\partial z^2}, \tag{49}$$

where $v_1=v_2=\sqrt{\lambda_x/n_0}$.

To derive integrable reductions of system (47), (48), and (49), we have to impose a set of restrictions to the physical parameters and time scale. We already used the assumptions that the phase velocities of the fields are equal to each other. One can use the same approximation of the one-directional propagation of wave, i.e., $\partial_z\approx -v^{-1}\partial_t+O(\epsilon)$, where ϵ is a small parameter. Introducing the variable $\partial_{\tilde{\chi}}=\partial_z+v^{-1}\partial_t$, we obtain the following reduced evolution equations:

$$\frac{\partial\mathcal{W}}{\partial\tilde{\chi}}=-\frac{n\hbar\omega_B f f_3}{v^2 n_0 g}\left(\frac{\hbar\omega_B}{f}+\mathcal{E}_{zz}\right)S_y, \tag{50}$$

$$\frac{\partial\mathcal{E}_{zz}}{\partial\tilde{\chi}}=\frac{n\hbar\omega_B f f_3}{v^2 n_0 g}\mathcal{W}S_y. \tag{51}$$

System (47), (50), and (51) is the second integrable system of evolution equations. From these equations the following integral can be derived:

$$\mathcal{W}^2+\left(\mathcal{E}_{zz}+\frac{\omega_B\hbar}{f}\right)^2=\mathcal{U}_0^2(t). \tag{52}$$

Using Eq. (52), we are able to present integrable system of equations (47), (50), and (51) in the dimensionless form

$$\partial_\chi\mathcal{E}=-b_1\mathcal{U}S_y, \tag{53}$$

$$\partial_\tau S_y=b_1\mathcal{U}S_x-\mathcal{E}S_z, \tag{54}$$

$$\partial_\tau S_x=-b_1\mathcal{U}S_y, \tag{55}$$

$$\partial_\tau S_z=\mathcal{E}S_y. \tag{56}$$

Where

$$\mathcal{E}(\chi,\tau)=\frac{\mathcal{W}(\chi,\tau)}{\mathcal{U}_0(t)},\quad \mathcal{U}(\chi,\tau)^2+\mathcal{E}(\chi,\tau)^2=1,$$

$$\chi=\tilde{\chi}\frac{n\omega_B f_3^2}{gn_0v^2},\quad \tau=\frac{f_3}{\hbar}\int_0^t\mathcal{U}_0(t')dt',\quad b_1=\frac{f}{f_3}.$$

Lax representation of system (53)–(56) has the following form:

$$\partial_\tau\Phi=\begin{pmatrix} -i\lambda\mathcal{U} & (\lambda+\beta)\mathcal{E} \\ -(\lambda-\beta)\mathcal{E} & i\lambda\mathcal{U}\end{pmatrix}\Phi, \tag{57}$$

$$\partial_\chi\Phi=\frac{b_1}{b_1^2-4\lambda^2}\begin{pmatrix} -i\lambda S_z & (\lambda+\beta)(b_1S_x-2i\lambda S_y) \\ (\beta-\lambda)(2i\lambda S_y+b_1S_x) & i\lambda S_z\end{pmatrix}\Phi, \tag{58}$$

where $\mathcal{U}^2+\mathcal{E}^2=1,\ \beta=\frac{1}{2}\sqrt{b_1^2-1},\ \lambda$ is the spectral parameter.

We will solve the problem on the entire axis for τ for the sufficiently fast decaying potential: $\mathcal{E}(\tau,\chi)\to 0,\ \tau\to\pm\infty$ and its derivatives. The spin system is assumed to be in an initially inverted state and asymptotically must tend to the stable ground state: $S_z(\tau,\chi)=-1,\ \tau\to\infty$.

We consider only the case of $|b|<1$. For this case, the involution property (27) is described by the matrix

$$\hat{M}=\begin{pmatrix} 0 & 1 \\ -1 & 0\end{pmatrix}. \tag{59}$$

The Jost functions corresponding to decaying for $\tau\to\pm\infty$ potential and its derivatives and ground state $\mathcal{E}(\tau,0)=0$ possess the following asymptotic:

$$\Phi^{\pm}=\exp(-i\lambda\sigma_3\tau),\quad \tau\to\pm\infty, \tag{60}$$

and may be written as

$$\Phi^{\pm}=\begin{pmatrix}\phi_1^{\pm} & -\phi_2^{\pm *}\\ \phi_2^{\pm} & \phi_1^{\pm *}\end{pmatrix}.$$

These functions are related by the scattering matrix $\hat{\mathcal{T}}$ by the relation

$$\Phi^{-}=\Phi^{+}\hat{\mathcal{T}}, \tag{61}$$

where

$$\hat{\mathcal{T}}=\begin{pmatrix}\mathcal{A}^* & \mathcal{B}\\ -\mathcal{B}^* & \mathcal{A}\end{pmatrix}. \tag{62}$$

Using the following presentation of the Jost functions

$$\Phi^{+}(\tau)=e^{-i\lambda\sigma_3\tau}+\int_{-\infty}^{\tau}\begin{pmatrix}\lambda\mathcal{K}_1(\tau,s) & (\lambda+\beta)\mathcal{K}_2(\tau,s)\\ -(\lambda-\beta)\mathcal{K}_2^*(\tau,s) & \lambda\mathcal{K}_1^*(\tau,s)\end{pmatrix}e^{-i\lambda\sigma_3 s}ds, \tag{63}$$

we derive from Eq. (63) and (57)

$$\mathcal{K}_2(\tau,\tau)[1+\mathcal{U}(\tau)]=\mathcal{E}(\tau)[1-i\mathcal{K}_1(\tau,\tau)]. \tag{64}$$

Using Eq. (64) and relation $\mathcal{U}^2+\mathcal{E}^2=1$, we obtain

$$\mathcal{E}(\tau)=\frac{2[1-i\mathcal{K}_1(\tau,\tau)]\mathcal{K}_2^*(\tau,\tau)}{[1+i\mathcal{K}_1^*(\tau,\tau)][1-i\mathcal{K}_1(\tau,\tau)]+|\mathcal{K}_2(\tau,\tau)|^2}. \tag{65}$$

Substituting the components of function (32) in Eq. (30) and integrating with the following weights:

$$e^{-i\lambda y}[2\pi(\lambda-\beta)]^{-1},\quad e^{-i\lambda y}(2\pi\lambda)^{-1},$$

we find the Marchenko equations ($y\leqslant\tau$)

$$\mathcal{K}_2^*(\tau,y)=\mathcal{F}_\beta(\tau+y)+i\int_{-\infty}^{\tau}\mathcal{K}_1(\tau,s)\partial_y\mathcal{F}_\beta(s+y)ds, \tag{66}$$

$$\mathcal{K}_1^*(\tau,y)=\int_{-\infty}^{\tau}\mathcal{K}_2(\tau,s)(\beta-i\partial_y)\mathcal{F}_0(s+y)ds, \tag{67}$$

where

$$\mathcal{F}_\beta(y)=\int_{\mathcal{C}_1}\frac{\mathcal{B}(\chi)}{\mathcal{A}(\chi)}\frac{e^{-i\lambda y}}{2\pi(\lambda-\beta)}d\lambda, \tag{68}$$

and $\mathcal{C}_1$ is the contour that passes along the real axis and above all poles in the upper half plane. $\mathcal{F}_0=\mathcal{F}_\beta(\beta=0)$. Condition of reality of E imposes some restrictions to the spectral data, e.g., poles may appear only in anticomplex conjugated pairs: $\lambda_1=-\lambda_2^*$.

Let us consider the initial-boundary conditions corresponding to the initially inverted spin system:

$$|\mathcal{E}(\tau,0)|=\text{const}\ll 1,\quad S_z(-\infty,\chi)=1,\quad S_x(-\infty,\chi)=0, \tag{69}$$

that is the same as Eq. (35).

Using problem (57), we find

$$\rho(0;\lambda)=\rho_0=\frac{\mathcal{B}(0;\lambda)}{\mathcal{A}(0;\lambda)}\approx\frac{-\lambda}{2}\int_{-\infty}^{\infty}\mathcal{E}(\tau,0)e^{2i\lambda\tau}d\tau. \tag{70}$$

For $\mathcal{E}(\tau,0)=$const, the scattering coefficient ρ_0 does not depend on λ. The dependence $\rho(\chi)$, we find using Eq. (58)

$$\rho(\chi)=\rho_0\exp\left(\frac{2ib_1\chi\lambda}{b_1^2-4\lambda^2}\right). \tag{71}$$

Using the above approximations, we find the following approximate solution to system (53)–(56):

$$\mathcal{E}(\tau,\chi)=E_2(\theta)\left[1+O\left(\frac{1}{\sqrt{\chi}}\right)\right], \tag{72}$$

where

$$E_2(\theta)=\frac{2Z\rho_0e^{\theta}}{Z^2+|\rho_0|^2e^{2\theta}},\quad \theta=2\sqrt{b_0\chi\tau},\quad Z=1-\frac{|\rho_0^2|}{4}e^{2\theta}.$$

As it follows from obtained solution (72) for large enough χ the amplitudes acoustic fields associated with avalanches for these two different geometries of interactions are described asymptotically by the close solutions. Indeed, solution (72) under approximations used here differs from Eq. (40) only by the phase factor. Differences between these solutions have an order of $\chi^{-1/2}$.

Solution (72) is depicted in Fig. 2 (solid line) together with the numerical solution of evolution equations (53)–(56) (dashed line). Numerical analysis shows that the asymptotic

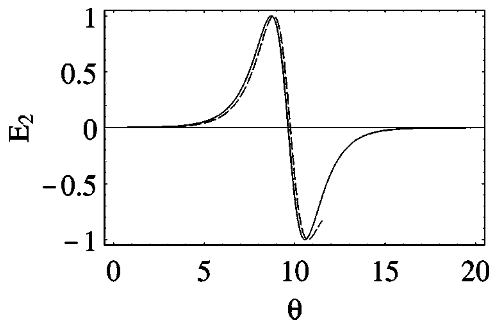

FIG. 2. Dependence of E_2 versus θ. Solution (72) is shown by the solid line. The numerical solution of system (53)–(56) is depicted by the dashed line.

solution (72) gives a good approximation for the leading front of the phonons avalanche for $-\ln|\rho_0|\gg 1$.

VI. THE QUASIMONOCHROMATIC APPROXIMATION

Here, we find the solution for the models that describe the dynamics of acoustic pulses of the order of or shorter than ω_B^{-1}. in duration by using the ISTM. These models are the most general integrable reduction of the original systems (11)–(14) and (47)–(49). However, it is also of interest to find other integrable reductions of this model that arise under additional assumptions. In general, these models are easier to solve and analyze. On the other hand, soliton solutions and other coherent structures arise in these models from the balance between dispersion, cross modulation, nonlinear mixing, etc. The corresponding terms in the equations simulate the real physical effects that show up at various field amplitudes and degrees of spin reversal in our problem. Therefore, it is important to determine the conditions when these effects, while being mutually balanced, give rise to solitons and other coherent structures. Studying these models is also useful for solving similar nonintegrable models because soliton and other stable solutions of integrable models can be used as a zero approximation in constructing the perturbation theory. As above, we use the condition of equal group velocities: $v_\perp = v_\parallel$. Let us now pass to quasimonochromatic fields in system (11)–(14):

$$\frac{\mathcal{E}_\perp G_\perp}{\hbar\omega_B} = \tilde{E}\exp[i(\omega t - kz)], \tag{73}$$

$$S_\perp = R\exp[i(\omega_B t - kz)], \quad |\omega_B - \omega| \ll \omega_B. \tag{74}$$

We use the slow-envelope approximation, which requires the satisfaction of the inequalities

$$\left|\frac{\partial\tilde{E}}{\partial t}\right| \ll \omega_B|\tilde{E}|, \quad \left|\frac{\partial\tilde{E}}{\partial z}\right| \ll k|\tilde{E}|,$$

$$\left|\frac{\partial R}{\partial t}\right| \ll \omega_B|R|, \quad \left|\frac{\partial R}{\partial z}\right| \ll k|R|. \tag{75}$$

Let us change to the variables

$$\tilde{z} = z\frac{knG_\perp^2}{v^2 n_0\hbar\omega_B}, \quad \tilde{\tau} = \left(t - \frac{z}{v}\right)\omega_B,$$

and denote

$$U = \frac{\mathcal{E}_\parallel G_\perp}{\hbar\omega_B}, \quad \tilde{E} = \frac{\mathcal{E}_\perp G_\perp}{\hbar\omega_B}, \quad \nu_0 = \frac{\omega_B - \omega}{2\omega_B}.$$

To simplify the description of the dynamics of the longitudinal field $\mathcal{E}_\parallel$, we use the condition of unidirectional field propagation. Given these approximations and changes, Eq. (11) take the form

$$\frac{\partial\tilde{E}}{\partial\tilde{z}} = iR, \tag{76}$$

$$\frac{\partial U}{\partial\tilde{z}} = -\frac{b_0}{2}\frac{\partial S_3}{\partial\tilde{\tau}}. \tag{77}$$

We find from Eqs. (76), (77), and (14) that the fields U and E are related by

$$U(\tilde{\tau},\tilde{z}) = -\frac{b_0}{4}|\tilde{E}(\tilde{\tau},\tilde{z})|^2 + U_1(\tilde{\tau}). \tag{78}$$

Here, $U_1(\tilde{\tau})$ is determined by the boundary conditions. Without loss of generality, we choose $U_1 \equiv 0$. Using equality (78), we reduce the Bloch equations (12)–(14) to

$$\frac{\partial R}{\partial\tilde{\tau}} = i\left(2\nu_0 - \frac{\beta_0^2}{4}\left|\tilde{E}\right|^2\right)R + i\tilde{E}S_3, \tag{79}$$

$$\frac{\partial S_3}{\partial\tilde{\tau}} = \frac{i}{2}(\tilde{E}^* S_\perp - \tilde{E}S_\perp^*). \tag{80}$$

As a result, we obtain the system of equations (69), (72), and (73), which is formally identical to our integrable system suggested previously [19]. This system was used to describe the generation and evolution of ultrashort electromagnetic light pulses in two-level optical media in the quasimonochromatic approximation. In Ref. [20], we found soliton and periodic solutions for this model. The Lax representations for the integrable system (76), (79), and (80) are

$$\partial_{\tilde{\tau}}\Phi = \begin{pmatrix} -i\lambda^2 - i\frac{\beta_1}{4}|\tilde{E}^2| & \gamma\tilde{E} \\ \tilde{\gamma}\tilde{E}^* & i\lambda^2 + \frac{\beta_1}{4}|\tilde{E}^2| \end{pmatrix}\Phi \equiv \mathbf{L}_2\Phi, \tag{81}$$

$$\partial_{\tilde{z}}\Phi = \frac{1}{4(\lambda^2+\nu_0)}\begin{pmatrix} -iS_3 & 2\gamma R \\ 2\tilde{\gamma}R^* & iS_3 \end{pmatrix}\Phi \equiv \mathbf{A}_2\Phi, \tag{82}$$

where

$$\gamma = \frac{1}{2}(\beta_1\lambda + i\sqrt{1+\beta_1^2\nu_0}),$$

$$\tilde{\gamma} = -\gamma^*, \quad \beta_1 = \frac{b_0^2}{2}.$$

Using the same quasimonochromatic approximations for fields

$$\frac{\mathcal{W}f_3}{\hbar\omega_B} = \tilde{W}\exp[i(\omega t - kz)] + \tilde{W}^*\exp[-i(\omega t - kz)], \tag{83}$$

$$S_x + iS_y = \tilde{S}\exp[i(\omega_B t - kz)], \quad |\omega_B - \omega| \ll \omega_B, \tag{84}$$

where

$$\left|\frac{\partial\tilde{W}}{\partial t}\right| \ll \omega_B|\tilde{W}|, \quad \left|\frac{\partial\tilde{W}}{\partial z}\right| \ll k|\tilde{W}|,$$

$$\left|\frac{\partial\tilde{S}}{\partial t}\right| \ll \omega_B|\tilde{W}|, \quad \left|\frac{\partial\tilde{S}}{\partial z}\right| \ll k|\tilde{W}|, \tag{85}$$

we derive from Eq. (47)–(49), the relation

$$\frac{f_3\mathcal{E}_{zz}}{\hbar\omega_B} = -\frac{f}{2f_3}|\tilde{W}|^2 + U_2(\tilde{\tau}). \tag{86}$$

Then, using the slow envelopes and the rotating wave approximation to system (47)–(49) and taking into account Eq. (86), we derive a system, which differs from Eqs. (76), (79), and (80) only by changing of the notation.

It can be easily shown that using the above quasimonochromatic approximations one derives the equations that are equivalent to system (76), (79), and (80) starting from systems (19)–(21) and (53)–(56).

Some information on the field dynamics can be obtained by analyzing the structure of this Lax pair and by comparing it with a similar Lax pair in Ref. [20]. The contribution of the longitudinal acoustic field shows up in the presence of terms with the coefficient β_1 in the matrices $\mathbf{L}_2$ and $\mathbf{A}_2$. As a result, for soliton and other solutions, including the longitudinal field manifests itself in a change of the pulse shape and in the appearance of a nonlinear phase addition of the order of $i\beta_1\int_0^{\tau}|\tilde{E}|^2 d\tilde{\tau}$. Since the contribution of the longitudinal field for a small ratio $G_{\parallel}/G_{\perp} \sim \epsilon$ is of the order of ϵ^2, its contribution to the dynamics of the transverse field in such media can be disregarded, which is attributabed to the quasimonochromatic approximation used above. In the case of ultrashort pulses considered above, i.e., for pulses $\tau_p \sim \omega_B^{-1}$ in duration, the contribution of the longitudinal field is the same in order of magnitude as that of the transverse field. This difference stems from the fact that Eqs. (76) and (77) describe the long-wave–short-wave resonance, which is much less effective than the short-wave resonance in the case considered in previous sections of this paper. We can conclude that the effects related to the coupling of transverse-and-longitudinal sound pulses are much more pronounced for ultrashort pulses than those in the quasimonochromatic limit. In deriving system (76), (79), and (80), we assumed the transverse field to produce rapid oscillations between Zeeman levels. In this case, the nonlinear effects are mainly attributabed to the interaction of the transverse field with a two-level medium. Consider the other extreme case where there are virtually no transitions between levels; i.e., the change in S_3 can be ignored. Applying the approximations used above, we obtain the following additional reduction of Eqs. (76), (79), and (80):

$$\frac{\partial\tilde{E}}{\partial\tilde{z}} = iR, \tag{87}$$

$$\frac{\partial R}{\partial\tilde{\tau}} = i\left(2\nu_0 - \frac{\beta_1}{2}|\tilde{E}|^2\right)R + i\tilde{E}. \tag{88}$$

This integrable system of equations can be reduced to the Thirring model by a simple gauge transformation. It also has stable soliton and other coherent solutions and can be analyzed in detail in terms of the ISTM (see, e.g., Ref. [21]). In this case, the existence of soliton and other coherent structures is attributabed to the nonlinear phase modulation produced by the longitudinal field. The transverse field manifests itself in establishing a coherent coupling between the field and the two-level medium in the linear limit.

VII. CONCLUSION

We find the integrable systems of evolution equations describing dynamics of acoustic fields in paramagnetic with spin 1/2 for two different geometries of interaction. Asymptotic solutions to these different systems corresponding to the phonon avalanches consist of the infinite trains of pulses. We find that the forms of the leading pulses tend asymptotically to the same form independently on the geometry of interaction. It is worth emphasizing that integrability of these models allows one to investigate the nonlinear stage of coherent evolution for more realistic physical parameters of sound pulses than in Ref. [11] and in related theoretical studies.

Estimate parameters of fields and medium required for observation of formation of the acoustic picosecond pulses. Consider, for instance, crystal of MgO containing paramagnetic impurities Fe^{2+} at the temperature $T=4$ K. Let the magnetic field strength be such that the Zeeman splitting is $\omega_B = 10^{12}$ s^{-1}. This corresponds to the realistic strength of the magnetic field. Coefficients of the medium are the following: [8], $G_\gamma \sim 10^{-13}$ erg, $n \sim 10^{19}$ cm^{-3}, $n_0 \sim 3-4$ g/cm^3, $v \approx 5-10\times 10^5$ cm/s, $\lambda_\gamma \approx 5\times 10^5 - 10\cdot 10^{11}$ din/cm^2. Under such conditions the peak intensity of the acoustic pulse can be $I \sim 10^8$ V/cm^2 and duration can be $\tau_p \sim 10$ ps.

Conditions of phonons avalanche observation are described in Refs. [13,14]. We demonstrate here that analogous avalanches can be observed even in the more common cases of transverse-and-longitudinal acoustic waves and for a few-cycle pulses. It is known that for picosecond time scale τ_p in some crystals losses associated with sound pulses propagations are proportional to τ_p^{-2}. For quasimonochromatic pulses with the carrying frequency ω_B durations are at least of the order of $0.1\omega_B^{-1}$. Therefore, for the same durations losses corresponding to a few-cycle pulses are at least 100 times less than that of a quasimonochromatic pulse.

Evolution equations close to those studied above may arise in another physical situation. Consider, for instance, phonon-assisted spin-flip transitions between the Zeeman sublevels for two different mesoscopic systems: GaAs quantum dots (localized states) and the two-dimensional electron system in the quantum Hall regime for filling factor 1 (delocalized states). In Ref. [22], it is shown that part of the Hamiltonian describing the spin-phonon interaction with the strain waves can be written in a form close to presented in the present paper. Following the above assumptions one can derive the evolution equations close to those studied here. The spin flip in such systems can be described by the soliton solution for $S_z(\tau,\chi)$, which can be derived from the Marchenko equations constructed by the way described in this paper.

ACKNOWLEDGMENTS

Research was supported in part by the interbranch integration project for the fundamental research of the Siberian Branch of RAS, a ''theoretical and experimental study of the production of the ordered heterostructures (photonic crystals), and its functional and nonlinear-optical structures,'' and the Russian Foundation of Basic Research (Grant No. 03-02-16297).

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